Statistical Model of Downlink Interference for the Performance Evaluation of CDMA Systems

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***Abstract*—A statistical model of the downlink interference for the analytical performance evaluation of code-division multiple-access (CDMA) systems is proposed. Based on *Wilkinson's* approximation, the statistical model of the relative other cell interference is developed in closed form. The best base station (BS) selection in the shadowed environment with a correlation is taken into consideration. We show that interference statistics obtained by the proposed analytical model agree with simulation results. It is also shown that the proposed analytical model can replace *Monte Carlo* simulation in outage analysis provided in [1].**

***Index Terms*—CDMA, downlink, interference, log-normal shadowing, the best BS selection.**

I. Introduction

THE received bit energy per noise power density (E_b/N_t) on the code-division multiple-access (CDMA) downlink is mainly influenced by the ratio of interference power from adjacent base stations (BSs) to that from the connected BS. Mobile station (MS) in a shadowed environment is connected to the BS with the smallest propagation loss, i.e., the best BS, rather than the closest BS. The best BS selection can considerably affect the statistics of the relative interference. Therefore, this effect should be considered in the performance evaluation of CDMA systems where cochannels are used in every cell.

The distribution of the relative interference depends both on relative distances from BSs and on the log-normally distributed random variables (RVs), is not tractable analytically. Though Viterbi [1] provided analytical methodologies for the outage analysis of both CDMA uplink and downlink, the interference statistics on downlink alone were obtained by *Monte Carlo* simulation. In another analytical approach, a regression method based on the simulation results was used for modeling the interference statistics on the CDMA downlink [2].

Since even the distribution of the sum of log-normal interferences at the given location is not known in closed form, log-normal approximation has been employed in many approaches [3]. However, the best BS selection was not considered in them. Although an exact analytical approach with the best BS selection was presented in [4], it is not easy to apply the provided distribution to the system performance analysis because it was not shown in closed form.

Manuscript received March 28, 2002. The associate editor coordinating the review of this letter and approving it for publication was Dr. Y.-D. Yao. This work was supported in part by the Ministry of Information and Communication of Korea - Support Project of University Foundation Research '99 supervised by IITA, and by the Korea Science and Engineering Foundation under Grant R01-1999-00205.

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Digital Object Identifier 10.1109/LCOMM.2002.804243

In this letter, based on *Wilkinson's* approximation that is one of the well-known log-normal approximations, a statistical model of the relative other cell interference is developed in closed form with considering the best BS selection. It is shown that the developed analytical model can replace *Monte Carlo* simulation in the outage analysis provided in [1].

II. Interference Distribution at the Given Location

The ratio of interference from the adjacent BSs to that from the connected BS at the given MS location in the shadowed environment, is to be statistically characterized.

Let us consider a cellular system where a number of BSs are distributed according to a pattern, e.g., hexagonal grid. MS of interest is connected to BS_0 at (r, θ), which are the polar coordinates with respect to BS_0. Let r_i be the distance from the BS_i, where $i = 0, 1, \ldots, M$. Note that $r = r_0$, and r_i for $i \neq 0$ can be represented by a function of (r, θ). Then, path gain is proportional to $L_i \equiv r_i^{-\gamma} 10^{(a\hat{\xi}+b\xi_i)/10}$, where γ is the path loss exponent. $\hat{\xi}$ and ξ_i are the MS shadowing and the BS_i shadowing, respectively. Both are *Gaussian* RVs with zero mean and standard deviation σ. In this model, site to site correlation is applied as $a^2 = 1 - b^2$ [1].

The relative interference Y can be defined as

$$Y \equiv \sum_{i=1}^{M} \frac{L_i}{L_0} = \sum_{i=1}^{M} \left(\frac{r}{r_i}\right)^{\gamma} 10^{b(\xi_i - \xi_0)/10} = \sum_{i=1}^{M} X_i \qquad (1)$$

where $X_i \equiv C_i 10^{b(\xi_i - \xi_0)/10}$ and $C_i \equiv (r/r_i)^{\gamma}$. The first and second moments of Y at the given position (r, θ) can be obtained as

$$m_{Y|r,\theta} \equiv E[Y|r,\theta] = \sum_{i=1}^{M} E[X_i|r,\theta] \qquad (2)$$

$$\nu_{Y|r,\theta} \equiv E\left[Y^2|r,\theta\right] = \sum_{i=1}^{M} E\left[X_i^2|r,\theta\right] + \sum_{j=1}^{M} \sum_{i=1, i\neq j}^{M} E[X_i X_j|r,\theta]. \qquad (3)$$

With the best BS selection, every MS is connected to the BS with the smallest propagation loss. Therefore, the inequality $L_0 > L_i$ should be met for all i in (1). If without the condition $L_0 > L_i$, the shadowing RVs of BS ξs are independent with one another. Then, their joint probability density function (pdf) is

$$f_{\underline{\xi}}(\underline{\xi}) = \prod_{i=0}^{M} f_{\xi_i}(\xi_i) = \prod_{i=0}^{M} \frac{1}{\sqrt{2\pi}\sigma} e^{-(\xi_i^2/2\sigma^2)} \qquad (4)$$

where $\underline{\xi} \equiv \{\xi_0, \xi_1, \ldots, \xi_M\}$. By using (4), the excepted value in (2) with the condition $L_0 > L_i$ can be computed as shown in (5) at the bottom of the page, where $A_i(x,y)$ is defined by using the error function $\mathrm{erf}(\cdot)$ as

$$A_i(x,y) \equiv \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{bx - 10\log C_i}{\sqrt{2}b\sigma} - y\frac{\sigma b \ln 10}{10\sqrt{2}}\right). \quad (6)$$

Note that C_i in the equations above depends on (r, θ). In similar way, the expected values in (3) can be derived as shown in (7) and (8) at the bottom of the page.

As the conventional *Wilkinson's* approximation shown in [3], we approximate Y as a log-normal RV. Let us define the decibel scaled value of Y as $Z \equiv 10\log Y$. Then Z becomes a Gaussian RV with mean $m_{Z|r,\theta} = 10\log(m^2_{Y|r,\theta}/\sqrt{\nu_{Y|r,\theta}})$ and variance $\sigma^2_{Z|r,\theta} = (100/\ln 10)\log(\nu_{Y|r,\theta}/m^2_{Y|r,\theta})$ [5]. Zorzi [4] provided the joint pdf of L_i with considering the best BS selection. From the provided formula, however, it is not easy to obtain pdf of Y or Z in closed from. In our approach, though the log-normal approximation is employed and numerical integrations are involved, the pdf of Z has been obtained in closed form of a normal distribution that would be very useful for the system performance analysis.

III. Interference Formula for Outage Analysis

We present an application example of the developed statistical model of the relative interference. The interference distribution obtained by *Monte Carlo* simulation for outage analysis in [1], is here derived by using the developed analytical model.

Outage probability on the CDMA downlink for speech services can be given by using the *Chernoff* bound [1]:

$$P_{\mathrm{out}} < \underset{s>0}{\mathrm{Min}}\, e^{(1+g)\rho\left(\frac{\lambda}{\mu}\right)\left(e^{sh}E\left[e^{sY}\right]-1\right)-s\beta\frac{W/R}{(E_b/N_t)_{REQ}}} \quad (9)$$

where g denotes a fraction of users in soft handover, ρ voice activity factor, λ/μ the offered load per cell or sector (in erlangs), h orthogonality factor, β a fraction of the total BS transmission power devoted to all speech channels, W spreading bandwidth, R data rate of speech channel, and $(E_b/N_t)_{\mathrm{REQ}}$ the required E_b/N_t for a target frame error rate.

In order to compute the outage probability above, we need a expected value of e^{sY}. As shown in [1], $E[e^{sY}]$ can be obtained by using the histogram of Y as follows:

$$E\left[e^{sY}\right] = \sum_{\{\mathbf{Y}\}} e^{s\mathbf{Y}} H(Y = \mathbf{Y}|\mathrm{BS}_0) \quad (10)$$

where $H(Y = \mathbf{Y}|\mathrm{BS}_0)$ is the histogram of Y for the users having been connected to BS_0. Viterbi [1] obtained this histogram by *Monte Carlo* simulation, but we get it by the developed model in the previous section. $H(Y = \mathbf{Y}|\mathrm{BS}_0)$ can be represented by

$$H(Y = \mathbf{Y}|\mathrm{BS}_0) = \iint_{A_0} \Pr(\mathbf{Y}_- \le Y < \mathbf{Y}_+|(r,\theta), \mathrm{BS}_0) f(r,\theta|\mathrm{BS}_0)\,dr\,d\theta \quad (11)$$

where A_0 denotes the entire region that is feasible for the selection of BS_0. $\Pr(\mathbf{Y}_- \le Y < \mathbf{Y}_+|(r,\theta), \mathrm{BS}_0)$ is the conditional probability given the position of (r, θ) and the selection of BS_0, where $\mathbf{Y}_- \equiv \mathbf{Y} - \Delta\mathbf{Y}/2$ and $\mathbf{Y}_+ \equiv \mathbf{Y} + \Delta\mathbf{Y}/2$. Since $Z \equiv 10\log Y$, this probability can be computed by using the Gaussian distribution for Z as follows:

$$\begin{aligned}\Pr(\mathbf{Y}_- \le Y < \mathbf{Y}_+|(r,\theta), \mathrm{BS}_0) &= \int_{10\log\mathbf{Y}_-}^{10\log\mathbf{Y}_+} f_Z(z|(r,\theta), \mathrm{BS}_0)\,dz \\ &= \frac{1}{2}\left[\mathrm{erf}\left(\frac{10\log\mathbf{Y}_+ - m_{Z|r,\theta}}{\sqrt{2}\sigma_{Z|r,\theta}}\right) - \mathrm{erf}\left(\frac{10\log\mathbf{Y}_- - m_{Z|r,\theta}}{\sqrt{2}\sigma_{Z|r,\theta}}\right)\right].\end{aligned} \quad (12)$$

$$\begin{aligned}E[X_i|r,\theta] &= \int C_i 10^{b(x_i-x_0)/10} f_{\underline{\xi}}\left(\underline{x}|L_0 > L_1, \ldots, L_0 > L_M\right) d\underline{x} \\ &= C_i \frac{\int_{-\infty}^{\infty}\left\{10^{-(bx_0/10)} f_{\xi_0}(x_0)\int_{-\infty}^{x_0-(10/b)\log C_i} 10^{bx_i/10} f_{\xi_i}(x_i)dx_i \cdot \prod_{n=1,n\ne i}^{M}\left[\int_{-\infty}^{x_0-(10/b)\log C_n} f_{\xi_n}(x_n)dx_n\right]\right\}dx_0}{\int_{-\infty}^{\infty} f_{\xi_0}(x_0)\prod_{n=1}^{M}\left[\int_{-\infty}^{x_0-(10/b)\log C_n} f_{\xi_n}(x_n)dx_n\right]dx_0} \\ &= C_i e^{(\sigma^2 b^2(\ln 10)^2/200)} \cdot \frac{\int_{-\infty}^{\infty} 10^{-(bx/10)} e^{-(x^2/2\sigma^2)} A_i(x,1)\prod_{n=1,n\ne i}^{M} A_n(x,0)dx}{\int_{-\infty}^{\infty} e^{-(x^2/2\sigma^2)}\prod_{n=1}^{M} A_n(x,0)dx}\end{aligned} \quad (5)$$

$$E\left[X_i^2|r,\theta\right] = C_i^2 e^{(\sigma^2 b^2(\ln 10)^2/50)} \cdot \frac{\int_{-\infty}^{\infty} 10^{-(bx/5)} e^{-(x^2/2\sigma^2)} A_i(x,2)\prod_{n=1,n\ne i}^{M} A_n(x,0)dx}{\int_{-\infty}^{\infty} e^{-(x^2/2\sigma^2)}\prod_{n=1}^{M} A_n(x,0)dx} \quad (7)$$

$$E[X_iX_j|r,\theta] = C_iC_j e^{(\sigma^2 b^2(\ln 10)^2/100)} \cdot \frac{\int_{-\infty}^{\infty} 10^{-(bx/5)} e^{-(x^2/2\sigma^2)} A_i(x,1)A_j(x,1)\prod_{n=1,n\ne i,n\ne j}^{M} A_n(x,0)dx}{\int_{-\infty}^{\infty} e^{-(x^2/2\sigma^2)}\prod_{n=1}^{M} A_n(x,0)dx} \quad (8)$$

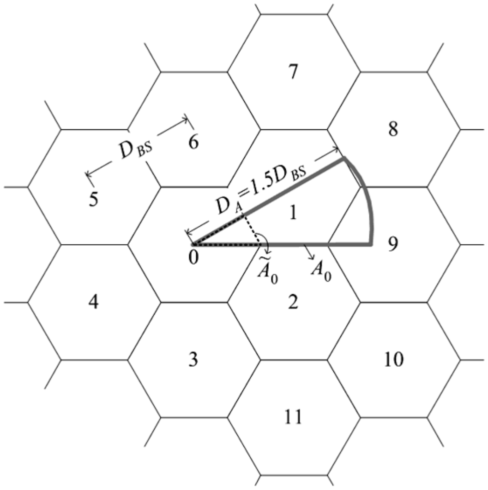

Fig. 1. Cell site deployment model.

In (11), $f(r, \theta|\mathrm{BS}_0)$ is the joint pdf of (r, θ) for the user having been connected to BS_0. With considering the best BS selection, this pdf can be derived as

$$f(r,\theta|\mathrm{BS}_0) = \frac{\Pr(\mathrm{BS}_0|r,\theta)f(r,\theta)}{\iint_{A_0}\Pr(\mathrm{BS}_0|r,\theta)f(r,\theta)drd\theta} \qquad (13)$$

where $f(r, \theta)$ is the joint PDF of (r, θ) on the region A_0 regardless of the connected BS. The probability of selecting BS_0 at the given position (r, θ) is computed as

$$\begin{aligned}\Pr(\mathrm{BS}_0|r,\theta) &= \Pr(L_0 > L_1, \ldots, L_0 > L_M|r,\theta) \\ &= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-(x^2/2\sigma^2)}\prod_{n=1}^{M}A_n(x,0)dx. \qquad (14)\end{aligned}$$

Consequently, $H(Y = \mathbf{Y}|\mathrm{BS}_0)$ can be obtained by using the equations above and the given distribution $f(r, \theta)$.

IV. Numerical Results

We compare the histogram $H(Y = \mathbf{Y}|\mathrm{BS}_0)$ obtained by the developed analytical model with that by *Monte Carlo* simulation. The result by the conventional *Wilkinson's* approximation is also presented and compared with them.

The cellular system is modeled by locating BSs at the centers of hexagonal grid pattern as shown in Fig. 1. An omni directional antenna pattern is used. To simplify the analytical procedure, the region A_0 is restricted to the sector of radius D_A and of angle $\pi/6$. Note that by symmetry, the relative position of users and BSs is the same throughout as for the sector of the figure. In the analytical model, we consider 11 BSs near BS_0, which are influential in the received other cell interference of users located on A_0 and connected toBS_0. The uniform distribution is applied to the user location. Therefore, $f(r, \theta) = 12r/\pi D_A^2$ for $0 < r \leq D_A$ and $0 \leq \theta < \pi/6$. The propagation parameter γ is set to 4, σ to 10 dB, and a^2 to 0.5.

We have also developed a computer simulation as in [1]. The *Monte Carlo* simulation model consists of 19 cells of two tiers, and we collect data from the center cell for statistics.

The histogram $H(Y = \mathbf{Y}|\mathrm{BS}_0)$ without considering the best BS selection is obtained by the conventional *Wilkinson's* approximation as given in [5]. In this case, a feasible region for the selection of BS_0 is limited to the triangle $\tilde{A}_0$ in Fig. 1 because MS selects the closest BS. A pure uniform distribution on $\tilde{A}_0$ should also replace $f(r, \theta|\mathrm{BS}_0)$ in (11).

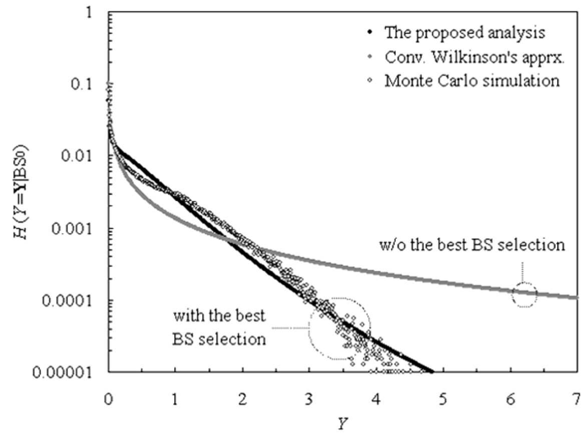

Fig. 2. Histogram of Y: $\Delta\mathbf{Y} = 0.01$.

Three $H(Y = \mathbf{Y}|\mathrm{BS}_0)$s obtained by the different methods above are shown in Fig. 2. The figure shows the difference between the results with and without the best BS selection. It is also shown the larger difference at the higher value of Y. It is therefore found that the best BS selection needs to be included for the exact interference analysis. In the figure, we can see that the analytical result by the proposed model agrees with the *Monte Carlo* simulation result. It is therefore confirmed that the proposed analytical model can be used with confidence, and also replace *Monte Carlo* simulation in the outage analysis provided in [1].

V. Conclusion

A statistical model of the shadowed downlink interference for the analytical performance evaluation of CDMA systems has been developed with considering the best BS selection. We have shown that interference statistics obtained by the proposed analytical model agree with simulation results. Moreover, the proposed model can replace *Monte Carlo* simulation in the outage analysis provided in [1]. Although we have shown only the example of speech service in this letter, the proposed model can be applied widely for the performance evaluation of CDMA systems including packet data service systems as shown in our previous work [6].

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